Computation of Longshore Energy Flux Using LEO Current Observations

by

Todd L. Walton, Jr.

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This report presents a computational technique for determining the longshore energy flux factor, $P_{ls}$, using current observations from the Littoral Environmental Observation (LEO) program. $P_{ls}$ is discussed in Chapter 4 of the Shore Protection Manual (SPM) (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977). The work was carried out under the coastal engineering research program of the U.S. Army Coastal Engineering Research Center (CERC).

The report was prepared by Dr. Todd L. Walton, Jr., Hydraulic Engineer, under the general supervision of Dr. J.R. Weggel, Chief, Evaluation Branch, Engineering Development Division.

Comments on this publication are invited.

Approved for publication in accordance with Public Law 166, 79th Congress, approved 31 July 1945, as supplemented by Public Law 172, 88th Congress, approved 7 November 1963.

TED E. BISHOP
Colonel, Corps of Engineers
Commander and Director
CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

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$^1$To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: $C = (5/9) (F - 32)$.

To obtain Kelvin (K) readings, use formula: $K = (5/9) (F - 32) + 273.15$. 
COMPUTATION OF LONGSHORE ENERGY FLUX USING LEO CURRENT OBSERVATIONS

by

Todd L. Walton, Jr.

I. INTRODUCTION

Prediction of sand transport rates along beaches is necessary to determine dredging quantities at inlets, effective life of various coastal structures such as jetties, and magnitude of erosion-accretion on beaches adjacent to inlets. Most computations of sand transport rate have previously been determined by computing a wave parameter dependent quantity termed the longshore energy flux factor \( P_{LS} \). Chapter 4 of the Shore Protection Manual (SPM) (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977) gives various equations for \( P_{LS} \) as a function of wave height, wave period, and wave angle with the shoreline at breaking. As wave angle is a difficult parameter to measure, an alternate approach is to use the longshore current as an independent quantity with which to determine \( P_{LS} \), since the wave angle with the shoreline is explicitly contained within the most acceptable formulas for longshore currents due to breaking waves (e.g., Longuet-Higgins, 1970). The present report incorporates the longshore current model (due to breaking waves) of Longuet-Higgins to determine the longshore energy flux factor, which in turn, can be used to estimate longshore sand transport rates.

II. DATA SOURCE

The computational technique in this report uses current observations from the Littoral Environmental Observation (LEO) program. The LEO program was developed by the Coastal Engineering Research Center (CERC) and is discussed by various investigators (Berg, 1969; Szuwalski, 1970; Bruno and Hiipakka, 1973; and Balsillie, 1975a). In the LEO program nearly simultaneous visual observations of breaker conditions (height, period, angle of approach, and type), local winds, longshore currents, rip currents, and beach geometry are made daily for a year or more. The selection of observation sites is not generally hindered by lack of access to the beach which often limits the use of instrumentation. Thus, depending on availability of trained observers, many sites along a considerable segment of shoreline may be established using LEO techniques.

The longshore current is estimated by measuring the shore-parallel distance and observing the direction that a sodium-fluoroscein dye packet injected into the surf (between the breakers and shore) travels in 1 minute. Observation of longshore current movement from the dye injections is representative of surface movement at the injection site, but may not always reflect the movement of water at depth or represent the average speed across the surf zone. As LEO measurements include the width of the surf zone as well as the distance from shore to the injection point of the dye, the longshore current can be treated as a point measurement on a spatially variable (across the surf zone) longshore current, the longshore current chosen in accordance with a theoretical profile having an assumed mixing constant. Balsillie (1975b) has shown that the LEO measurements of longshore currents (across surf zone) correlate very well with longshore currents calculated by the theoretical formula of Longuet-Higgins (1970).
III. DETERMINATION OF LONGSHORE ENERGY FLUX FACTOR

The following equation is equivalent to equation (4-28) in the SPM when calculating the longshore energy flux factor,

\[ P_{LB} = \frac{\rho g H_b W V_{LEO} C_f}{\left(\frac{5\pi}{2}\right) \left(\frac{V}{V_0}\right)_{LH}} \]  

(1)

where

- \( \rho \) = fluid density
- \( g \) = acceleration of gravity
- \( H_b \) = breaking wave height
- \( W \) = width of surf zone
- \( V_{LEO} \) = average longshore current due to breaking waves
- \( C_f \) = friction factor (assume 0.01)

and

\[ \left(\frac{V}{V_0}\right)_{LH} = 0.2 \left(\frac{X}{W}\right) - 0.714 \left(\frac{X}{W}\right) \ln \left(\frac{X}{W}\right) \]  

(2)

where \( X \) is the distance to dye patch from shoreline and \( \left(\frac{V}{V_0}\right)_{LH} \) is the Longuet-Higgins dimensionless longshore current velocity for an assumed mixing coefficient, \( P = 0.4 \), which agrees reasonably well with laboratory data (see Longuet-Higgins, 1970). The derivation of equation (1) is presented in the Appendix, as well as reference to equation (2).

It should be noted that as previous calculation equations for \( P_{LB} \) are based on significant wave heights (e.g., Ch. 4 in the SPM) equation (1) should also use significant wave height for breaking wave height. The recorded value of \( H_b \) in the LEO observation program is a reasonable approximation to significant breaking wave height. It should also be noted that as the LEO current observations are time-averaged, computing \( P_{LB} \) by the present method may provide a lower value of the longshore energy flux factor than given by equations based on significant breaking wave height to higher powers such as those in Chapter 4 of the SPM.

IV. EXAMPLE PROBLEM

GIVEN: A LEO observation with the following measured values of wave height, longshore current velocity, width of surf zone, and distance of dye patch from the shoreline

\[ H_b = 3.0 \text{ feet (0.91 meter)} \]
\[ V_{LEO} = 0.5 \text{ foot (0.15 meter) per second} \]
\[ W = 150 \text{ feet (45.7 meters)} \]
\[ X = 50 \text{ feet (15.2 meters)} \]
FIND: Longshore energy flux factor, $P_{ls}$

SOLUTION:

(a) Using equation (2) calculate $V/V_{ol}^{LH}$

$$\left(\frac{V}{V_{o}}\right)^{LH} = 0.2 \left(\frac{50}{150}\right) - 0.714 \left(\frac{50}{150}\right) \ln \left(\frac{50}{150}\right) = 0.33$$

(b) Now, using equation (1) calculate $P_{ls}$.

$$P_{ls} = \frac{64(3)(150)(0.5)(0.01)}{(\frac{5\pi}{2})(0.33)} = 55.3 \text{ pounds (25.1 kilograms) per second}$$

(c) The value of $P_{ls}$ corresponds to a sediment transport rate of 415,000 cubic yards (317,310 cubic meters) per year using the SPM equation (4-40) ($Q = 7.5 \times 10^3 P_{ls}$ in feet-per-second system).

(d) Annual average sediment transport rates for any field site would be estimated from LEO with a $P_{ls}$ value obtained by averaging the $P_{ls}$ values computed for each observation by the above method.
LITERATURE CITED


APPENDIX

DERIVATION FOR LONGSHORE ENERGY FLUX FACTOR

Derivation of equation (1) for longshore energy flux factor:

(a) From Longuet-Higgins (1970)

\[ V_b = \frac{5\pi}{8} \left( \frac{\kappa \beta}{C_f} \right) \left( g \frac{d_b}{d^j} \right)^{1/2} (m \sin \alpha_b \cos \alpha_b) \]  
(A-1)

where

\[ V_b = \text{longshore current at breaking zone} \]
\[ \beta = \text{a mixing parameter} \]
\[ d_b = \text{breaking depth} \]
\[ m = \text{beach slope} \]
\[ \alpha_b = \text{breaking wave angle} \]
\[ \kappa = \text{ratio of breaking wave amplitude to water depth} \]

(b) Using relationship \( 2\kappa = \frac{H_b}{d_b} \) equation (A-1) becomes

\[ V_b = \frac{5\pi}{16} \left( \frac{\kappa \beta}{C_f} \right) \left( \frac{1}{2\kappa} \right)^{1/2} m \left( gH_b \right)^{1/2} \sin 2\alpha_b \]  
(A-2)

(c) Longshore velocity at any point within surf zone can be defined as

\[ V = V_b \left( \frac{V}{V_b} \right) \left( \frac{V}{V_0} \right) \]  
(A-3)

where \( V \) is longshore current within surf zone and \( V_0 \) is theoretical longshore velocity at breaking, no mixing.

(d) From equation (58) of Longuet-Higgins (1970)

\[ \frac{V_0}{V_b} = \frac{1}{\beta} \]  
(A-4)

(e) Using equations (A-4), (A-3), and (A-2), longshore velocity is

\[ V = \left( \frac{V}{V_0} \right) \left( \frac{5\pi}{16} \right) \left( \frac{\kappa \beta}{C_f} \right) \left( \frac{1}{2\kappa} \right)^{1/2} m \left( gH_b \right)^{1/2} \sin 2\alpha_b \]  
(A-5)

(f) Using the SPM equation (4-28)
\[ p_{LS} = \frac{\rho g H_b^2}{16} C_{gb} \sin 2\alpha_b \]  

(A-6)

where \( C_{gb} \) equals group wave celerity equals \((g d_b)^{1/2}\) linear wave theory; therefore

\[ p_{LS} = \frac{\rho g H_b^2}{16} (d_b H_b)^{1/2} (gH_b)^{1/2} \sin 2\alpha_b \]  

(A-7)

(g) Using equation (A-2), (A-5), and (A-7) and assuming \( m = d_b/W \)

\[ p_{LS} = \frac{\rho g H_b^2 W V C_f}{\left(\frac{5\pi}{2}\right) \left(\frac{V}{V_0}\right)} \]  

(A-8)

(h) The value of \((V/V_0)\) can be assumed equal to that given by Longuet-Higgins (1970)

\[ \left(\frac{V}{V_0}\right) = \left(\frac{V}{V_0}\right)_{LH} \]  

(A-9)

(i) The value of \( V \) is measured using LEO technique

\[ V = V_{LEO} \]  

(A-10)

(j) Equation (A-8) now becomes

\[ p_{LS} = \frac{\rho g H_b^2 W V_{LEO} C_f}{\left(\frac{5\pi}{2}\right) \left(\frac{V}{V_0}\right)_{LH}} \]  

(A-11)
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Includes bibliographical references.

Appendix: Derivation for longshore energy flux factor.

A computational technique is presented for the longshore energy flux factor, $F_{LEO}$, using current observations from the Littoral Environment Observation (LEO) program. Chapter 4 of the Shore Protection Manual (SPM) gives various equations for $F_{LEO}$ as a function of wave height, wave period, and breaking wave angle. The present report details how $F_{LEO}$ can be calculated using longshore current and breaking wave height data only.

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Includes bibliographical references.
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A computational technique is presented for the longshore energy flux factor, $P_{LO}$, using current observations from the Littoral Environment Observation (LEO) program. Chapter 4 of the Shore Protection Manual (SPM) gives various equations for $P_{LO}$ as a function of wave height, wave period, and breaking wave angle. The present report details how $P_{LO}$ can be calculated using longshore current and breaking wave height data only.
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